Experiment: measure of the elastic deformation in three point bending

1. Introduction, general description

The proposed experiment aims at evaluating the elastic line of a beam in in three point bending: this corresponds to having the beam on two supports and loaded by a transverse force at the mid-span.
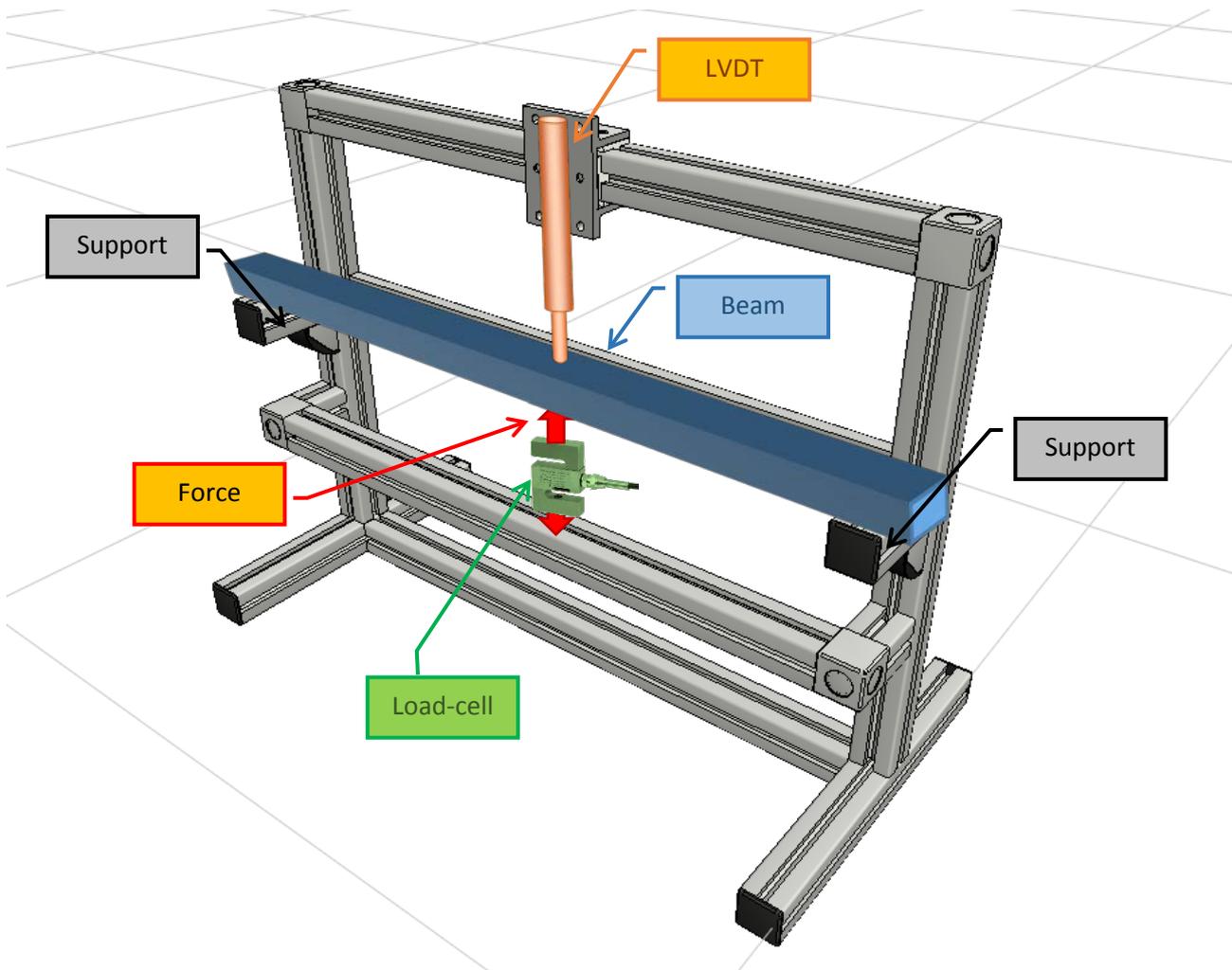

Fig. 1 – Loading frame for the experiment

The measure of displacement will be made by means of a displacement transducer of the type LVDT (*Linear Variable Differential Transformer*. The measure of the force will be made by means of a strain-gage load-cell (where the force is first converted in mechanical strain which in turn is measured with a bridge of electrical resistance strain-gages).

The force, measured by the load-cell, will be generated by means of a screw manually actuated. The screw will produce the force that will pass through the load-cell (where it will be measured) and will be applied to the center point of the beam

The displacement transducer is, instead, moveable along a direction parallel to the beam axis (longitudinal direction), to allow for the measurement at different locations: by measuring in a sufficient number of points (5 is sufficient for an adequate evaluation), at an adequate distance between them, it is possible to approximate the elastic line.

The measurements of the values of force and displacement will be performed by means of a two channels (one for the force, the other for the displacement) data acquisition board connected to the PC. A program developed in NI LabView® will be used to get the measures of force and displacement. The values will be then recorded (on paper or in electronic form) for subsequent examinations and to prepare the evaluation document to be completed in the provided tab on each own personal page of the Educational Portal.

Note: measures of force and displacement will be acquired simultaneously in several points along the beam length. Their ratio is the *point static stiffness* (measured in N/mm, inverse of the point compliance) that is, the value of stiffness necessary to obtain a unit displacement force (the force in N to obtain a displacement of 1 mm).

1.1. Procedure

The following 5 measuring points will be considered, they are limited to half beam for symmetry reasons:
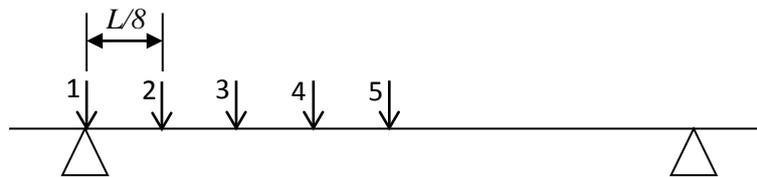
The experiment will be developed as follows (after the preparation of the setup, transducers, and connections):

a) Positioning of the LVDT in one of the measuring points along the beam axis
b) Measurement of the longitudinal position with a graduated scale in mm (or any device to measure the point position)
c) Application of the force by means of the screw device (loading from zero up to a value adequate to measure the force-displacement curve on video), then unloading, then repeating the loading-unloading cycle at least 2 times.
d) Recording of the point static stiffness, that is the force in N to obtain a unit displacement (1 mm)
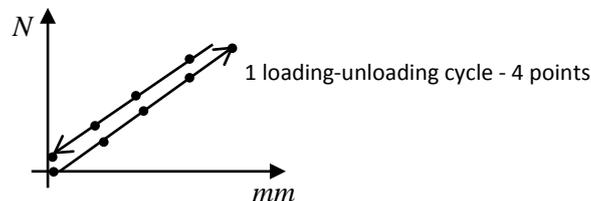
e) Repetition of steps from a) to d) for other four positions as much as possible equally spaced between one of the supports and the mid span of the beam

1.2. Materials provided for the experiment

The materials and tools provided are as follows (Fig. 2):

- Aluminum beam, cross section dimensions and distance between hinges to be measured
- Loading frame with supports, screw loading device, load-cell (AEP model TCA , 500 N force range), LVDT (AML/EU/±5/S, ±5 mm stroke range) and sliding support for its positioning
- Connecting cables with plugs

- Regulated stabilized laboratory power supply (ISO-TECH IPS303DD, Volt 0 → 30 V)
- Two-channels, USB data acquisition board with A/D conversion NI 9218 (in chassis cDAQ-9171, CompactDAQ, 1 slot, USB connected)
- Personal computer (PC) with dedicated data acquisition software (developed in the NI LabView® environment)
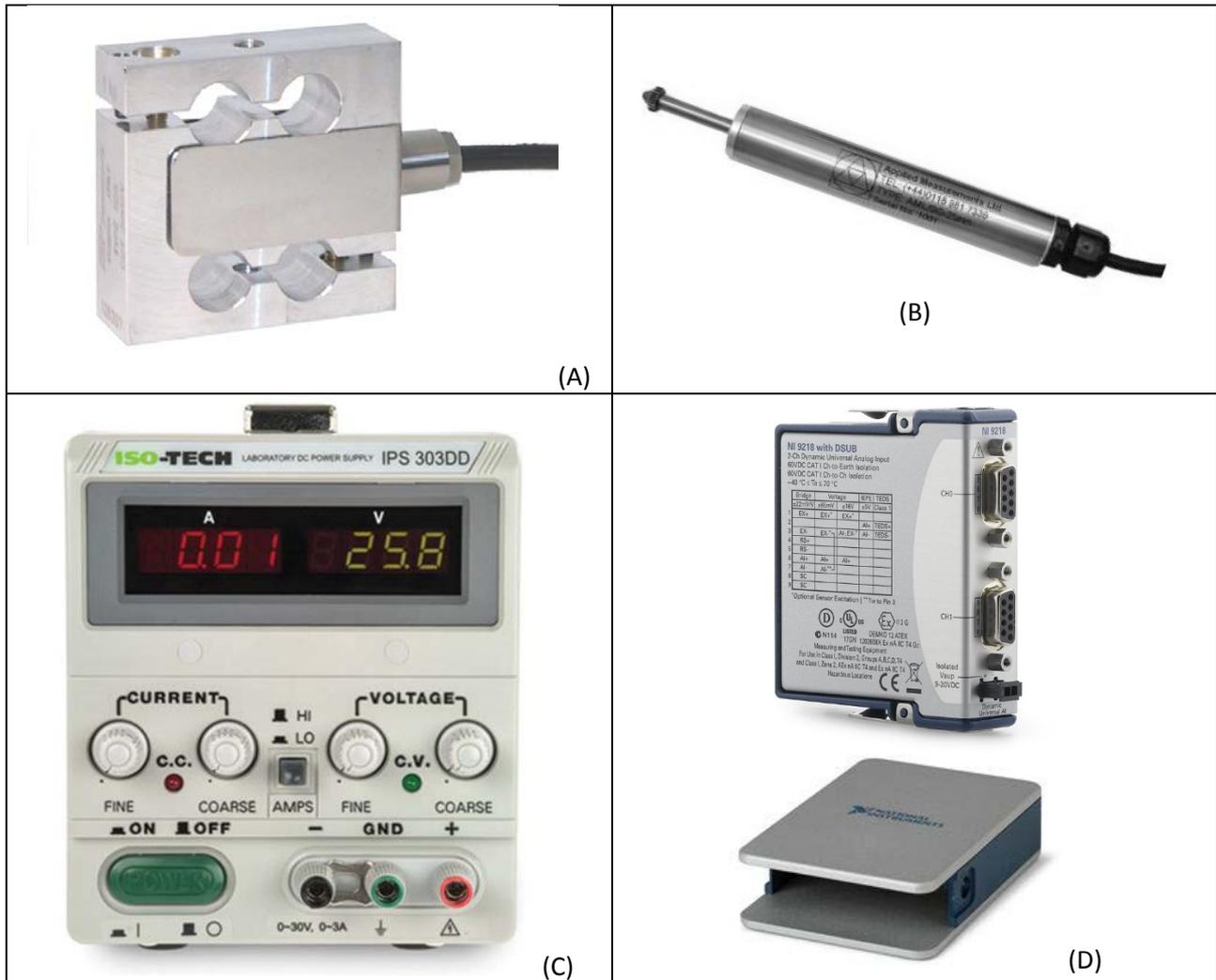- Caliper
- Ruler

Fig. 2 – Equipment for the experiment: (A) load-cell AEP model TCA; (B) LVDT; (C) power supply ISO-TECH IPS303DD; (D) Data acquisition board NI 9218 and chassis cDAQ-9171 (CompactDAQ)

2. Operation

First of all it is necessary to power on the PC and all the required electronic equipment. After that a login to the PC is required by using each own personal credentials of the Educational Portal (https://login.didattica.polito.it/secure/ShibLogin.php).

1. Switch the power supply on and set the output to a value slightly greater than 12 V (tentatively within 13 V and 15 V) using the knobs VOLTAGE (Fig. 2(C); if voltage read on the display on the right does not increase, raise the current limit with the knobs CURRENT)

2. Check connections or connect the power cables of the LVDT (from the power supply to the CompactDAQ) **by respecting the red/black colors** (the red plug shall be inserted in the red outlet, the black plug into the black outlet)
3. Check the connection of the LVDT to the CompactDAQ acquisition board (*)
4. Check the connection of the load cell to the CompactDAQ acquisition board (*)
5. Check the connection of the CompactDAQ acquisition board to the PC (*)
6. Place the beam on the supports and connect (mechanically) the loading device to the beam (if not already mounted)
7. Activate the acquisition program (Fig. 3) by means of the button available in the desktop dedicate directory
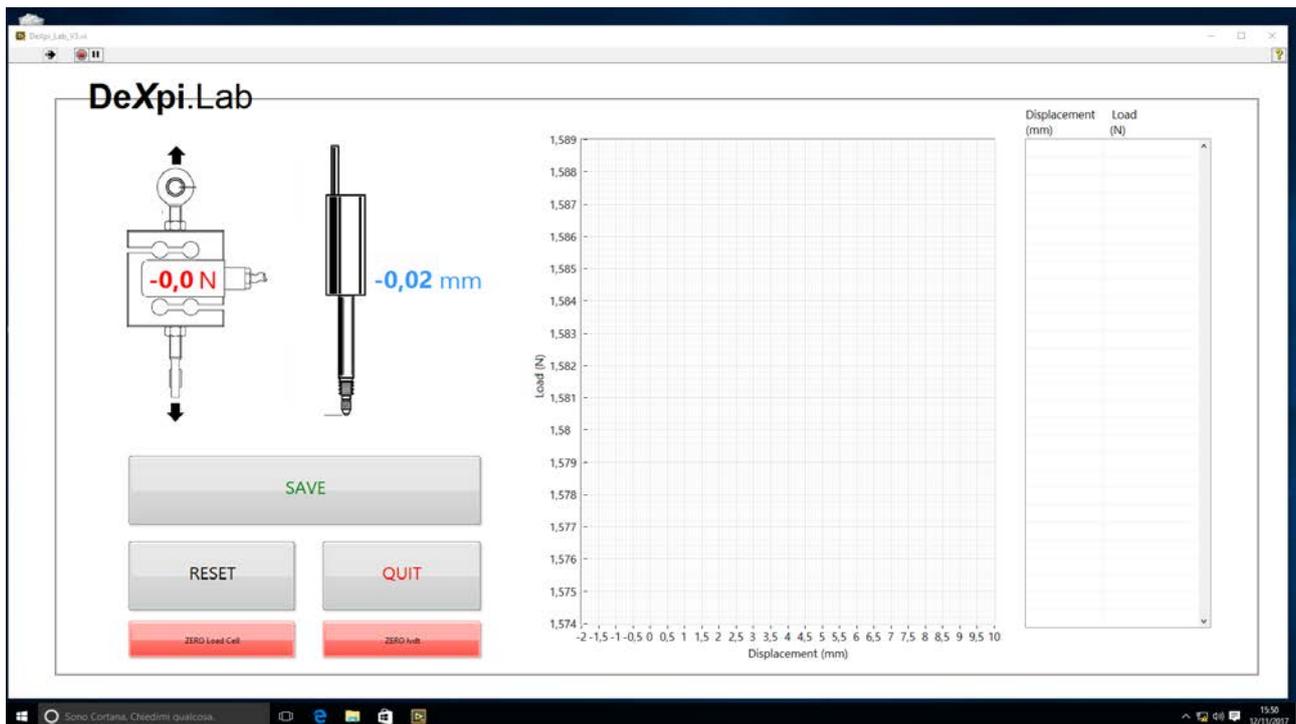

Fig. 3 – Screenshot of the acquisition software

8. Position the sliding support in the first position for the measure, at mid-span and zero the measure of load cell by means of the red button ZERO Load Cell
9. Position the longitudinal slide which supports the LVDT in the position in which the measure will be run. If the measure is to be done in the middle of the beam, then position the tip of the LVDT aside or on the top of the loading devide plate.
10. Fasten the handle to block the position of the LVDT positioning slide, to avoid measuring errors.
11. Regulate the vertical position of the LVDT to compensate following beam deformation. This means preloading the LVDT so that the unloaded value of displacements is 2-3 mm.
12. Zero the LVDT by means of the red button ZERO lvdt
13. Start the acquisition program
14. Start the test by loading and unloading the beam (at least 2 times to check for repeatability on the chart visualized on screen) following this procedure:
    14.1 Preload the beam with 20-30 N by means of rotating the hexagonal nut clockwise.
    14.2 Record the first loading and displacement values value either reading it on screen and recording it in a table (on paper or in electronic format) or by storing it by pressing the SAVE button

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14.3   Increase the force by acting on the loading handle until a increment of load of 20 N is obtained.

14.4   Read and store this second value (as done in step 13.1)

14.5   Increase again the load of a second rise for a further increase of 20 N.

14.6   Repeat the previous step up to a total load increment of 100 N (approximately). Store the value of the maximum displacement

14.7   Repeat steps 14.2-14.5 but decreasing the load of 20 N at a time, down to preload value.

14.8   Unload the beam to 0 N

14.9   Repeat the procedure 14.1-14.14.8 at least 1 more time.

14.10   If the values measured with the acquisition software are stored, step by step (using the SAVE functions) it is possible to export them as shown in Fig. 4 (right click on the measurements table, Export → Export Data To Excel)

10. Repeat the test, steps 13.1 to 13.7, by moving longitudinally the slider with the LVDT at the other four measurement positions between mid-span and one of the two supports. In these other points, do not exceed the maximum load P_{max} even if the maximum displacement will be less than the 5 mm measured at mid-span. Before storing new measurements, erase previous measurements by pressing the RESET button after step 13.8

Once the test finished leave everything in place: the laboratory assistants will then check and dismount the equipment if necessary.

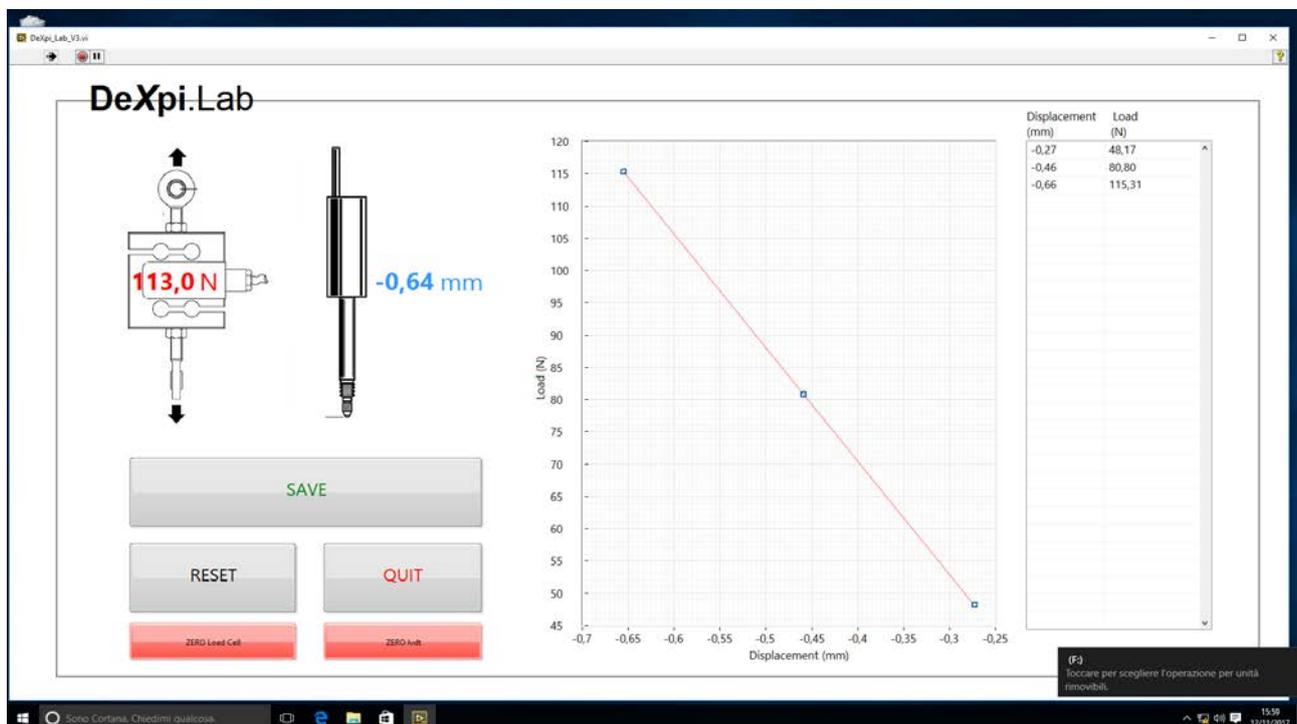**(*) in case of problems do contact the laboratory assistants**

Fig. 4 –Exporting the measurements

2.1. Evaluation of the local static (bending) stiffness

To compute the local static stiffness value of the beam, at each measurement point, a linear regression of the force values with respect to the displacement of force must be done. The slope of force-displacement

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line (chart in Fig. 4) is the local static stiffness value of the beam (in N/mm; the point static compliance is its reciprocal).

Linear regression can be done by means of whatever statistical tool, including the well-known function implemented in Excel (*REGR.LIN()* or *LINEST()* in the Italian or English versions of the program, respectively) or with other spreadsheet programs.

Warning: it is not correct to evaluate the stiffness (or the compliance) as the simple ratio of a single displacement and of the corresponding force. A bias error (inaccurate zeroing of one of the measured signals) implies an error in the ratio (as in Fig. 4).

3. Analysis of the experimental results

The results can be used to draw the elastic bending line. To do this, it is necessary to compute, in the measurement points, the displacement corresponding to a fixed value of force: this can be done by exploiting the point static stiffness values experimentally evaluated. Once the reference force defined (for convenience the maximum measured force can be used, otherwise another value) the corresponding displacements, in every point, can be calculated by dividing the value of the reference force (in N) by the local static stiffness value (in N/mm)

The values can be then reported in graphical form with any scientific drawing tool.

The experimental results can then be compared with prediction of the elastic line of a beam in three point bending according to the De Saint Venant theory (see Appendix A). The transverse displacement *v* with respect to the undeformed mid line can be estimated by the expression:

$$\left| v(z) \right| = \frac{P}{EJ}\left(\frac{zL^2}{16} - \frac{z^3}{12} \right) \qquad 0 \le z \le L/2$$

The maximum displacement at mid span *f* can be evaluated with the well-known expression:

$$\left| v\left(z = \frac{L}{2}\right) \right| = \frac{P}{EJ}\left(\frac{L^3}{48} \right) = f$$

Note that the theoretical point static stiffness *k(z)* can be evaluated as:

$$k(z) = \frac{EJ}{\left(\dfrac{zL^2}{16} - \dfrac{z^3}{12} \right)} \qquad 0 \le z \le L/2$$

Where:

- *P*, force applied at mid span
- *L*, distance (span) between the supports (the total beam length must be greater than this value and it is irrelevant for the current analysis)
- *E*, elastic modulus of the material (aluminum, about 70×10³ MPa)
- *J*, inertia moment of the beam section (measure the section size or ask the laboratory assistant if not given)
- *z*, longitudinal coordinate measured from one of the supports
- *v(z)*, transverse displacement at coordinate *z*
- *f*, center point displacement

4. Check of the experimental results

Verification of the accomplished experiment and of the correctly collected data will be done through the generation of an automated report through a form in each own personal page of the educational portal.

Attached, a facsimile (Appendix B) of the reporting scheme as it will be found.

5. Appendix A
Theory of the elastic line

Under the action of loads, either distributed or concentrated, transverse with respect to the beam axis, a distribution of shear and bending moment occurs. The actions of the bending moment and, with minor effect, of the shear cause translation and rotation of the points along the beam axis. The deformed shape of the axis of the beam is known as the elastic line. In the case of a beam with linear axis the result will be a folding of the axis with translation with respect to the initially linear axis.

From the theory of the elastic beams, it is possible to obtain that the local value of the curvature of the axis is proportional to the value of the bending moment in the considered point:

$$\chi_x(z) = \frac{M_x(z)}{E(z)J_x(z)}$$

With:

$\chi_x(z)$, curvature of the axis of the beam in the point with longitudinal coordinate z

$M_x(z)$, bending moment acting in the point with longitudinal coordinate z

$E(z)$, elastic modulus of the material of the beam in the point with longitudinal coordinate z

$J_x(z)$, quadratic moment of the section in the point with longitudinal coordinate z

For the sake of simplicity, the analysis is restricted to the bending on one plane only, namely plane yz. Moreover, the study is restricted to the case of constant elastic modulus and constant section equal to E and J_x respectively.

By neglecting the effect of the shear deformation, in the Euler-Bernoulli hypothesis, the curvature is equal to the first derivative of the section rotation in the considered point, and, for small rotations, equal to the second derivative of the transverse displacement $v(z)$:

$$\frac{d^2v(z)}{dz^2} \approx \chi_x(z) = \frac{M_x(z)}{EJ_x}$$

By double integration of the distribution of the bending moment, and applying the required boundary conditions, it is possible to evaluate the distribution of the transverse displacement $v(z)$ that is, the sought elastic line.

To solve the problem it is necessary first to obtain the bending moment distribution along the axis, then by double integration the elastic line will be calculated. By exploiting the symmetry of the problem (load at mid span and symmetrical supports) it is possible to restrict the analysis to one half of the structure only.

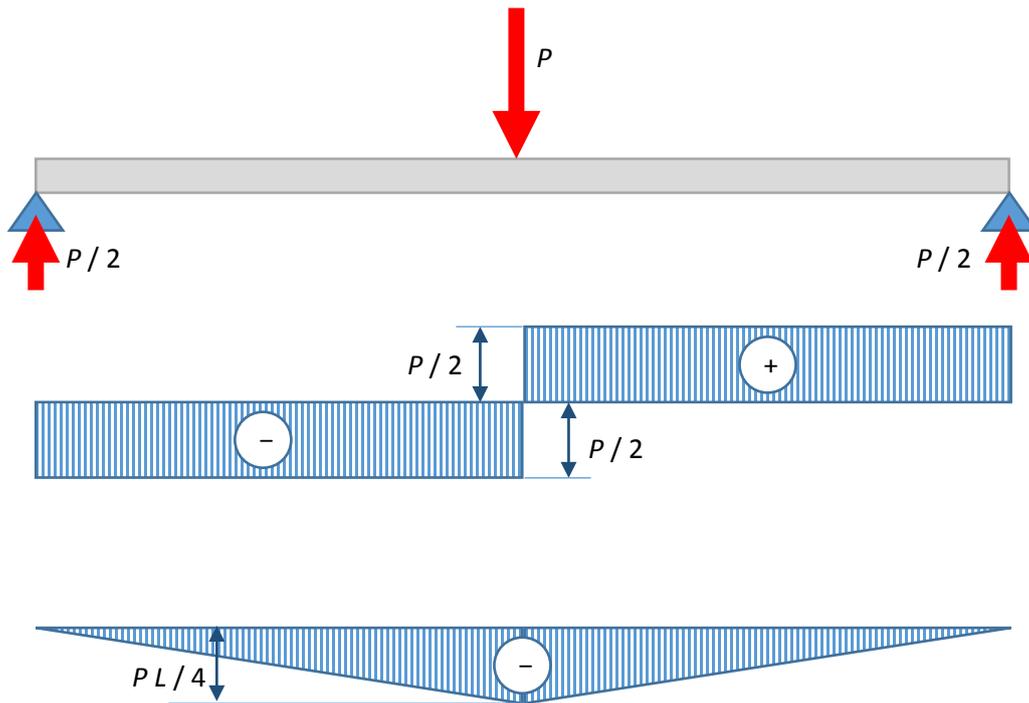Distributions of shear and bending moment are reported in Fig. A1

Fig. A1 – Distributions of shear load and bending moment

The distributions of shear load and bending moment are described by the following expressions:

$$T_y(z) = \left(\frac{dM_x(z)}{dz} \right) = \begin{cases} \dfrac{P}{2} & 0 \le z < \dfrac{L}{2} \\[2mm] -\dfrac{P}{2} & \dfrac{L}{2} < z \le L \end{cases}$$

$$M_x(z) = \begin{cases} \dfrac{P}{2}z & 0 \le z < \dfrac{L}{2} \\[2mm] -\dfrac{P}{2}z & \dfrac{L}{2} < z \le L \end{cases}$$

$$M_{x,\max} = M_x\left(z = \frac{L}{2}\right) = \frac{PL}{4}$$

It is necessary then to integrate the equation of the bending moment within the interval $0 \le z \le L/2$, imposing the following boundary conditions:

$$\frac{dv}{dz}\left(z = \frac{L}{2}\right) = 0$$
$$v(z = 0) = 0$$

That is, zero rotation at mid-span ($z = L/2$), and zero displacement at the support ($z = 0$).

First integration gives:

$$\frac{d^2 v(z)}{dz^2} = \frac{M_x(z)}{EJ_x} \quad \rightarrow \quad \int \frac{d^2 v(z)}{dz^2} = \int \frac{M_x(z)}{EJ_x} = \int \frac{Pz}{2EJ}\, dz$$

$$\frac{dv(z)}{dz} = \frac{Pz^2}{4EJ_x} + C_1$$

Imposing the first boundary condition (dv /dz (L / 2) = 0):

$$\left.\frac{dv(z)}{dz}\right|_{z=L/2} = \frac{PL^2}{16EJ_x} + C_1 = 0 \quad \rightarrow \quad C_1 = -\frac{PL^2}{16EJ_x}$$

Second integration gives:

$$\frac{dv(z)}{dz} = \frac{Pz^2}{4EJ_x} - \frac{PL^2}{16EJ_x} \quad \rightarrow \quad \int \frac{dv(z)}{dz} = \int \left(\frac{Pz^2}{4EJ_x} - \frac{PL^2}{16EJ_x} \right) dz$$

$$v(z) = \frac{Pz^3}{12EJ_x} - \frac{PL^2 z}{16EJ_x} + C_2$$

Imposing the second boundary condition ($v(z = 0) = 0$):

$$v(z = 0) = \frac{P0^3}{12EJ_x} - \frac{PL^2 0}{16EJ_x} + C_2 = 0 \quad \rightarrow \quad C_2 = 0$$

The final expression is therefore:

$$v(z) = \frac{P}{EJ_x}\left(\frac{z^3}{12} - \frac{L^2 z}{16} \right)$$

A graphical representation of the elastic line for the examined case is reported in Fig. A2.
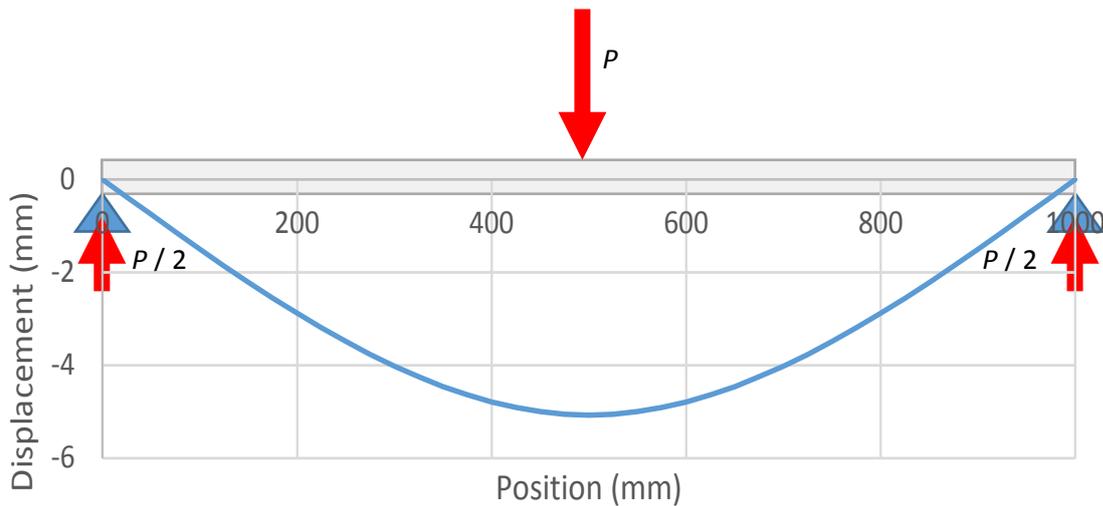

Fig. A2 – Elastic line of the beam in three point bending (not on scale: the numerical values are only for indication)

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6. Appendix B
Form for the experimental test reporting

Course: Fundamentals of Strength of Materials

Topic: Experimental analysis of the elastic deformation of the three point bending of a beam

Description of the experiment (mandatory field, 500 characters minimum)

Description of the procedure (mandatory field, 500 characters minimum)

Description of the used materials, tools and devices (mandatory field, 2000 characters minimum)

Findings (mandatory field, with automatic check of the results)

Point No.	1	2	3	4	5
Position, z (mm)	z_1	z_2	z_3	z_4	z_5
Transverse displacement, f (mm)	f_1	f_2	f_3	f_4	f_5
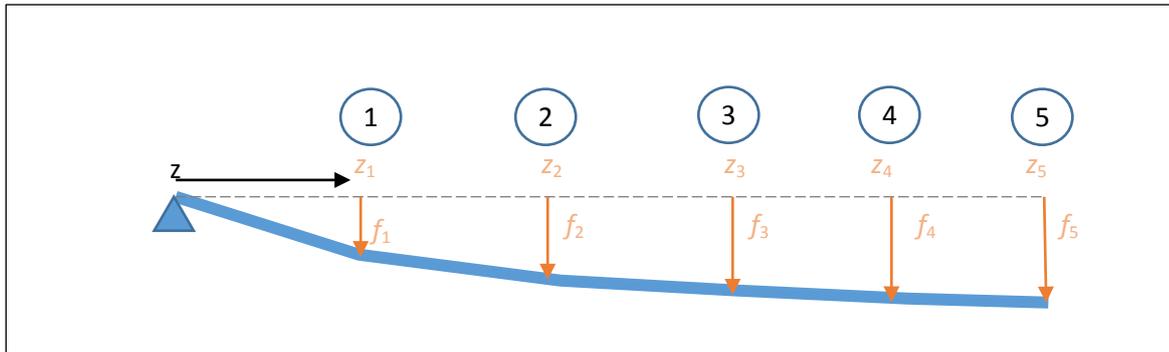
Results analysis and conclusions (mandatory field, 300 characters minimum)

Comments (optional field, 1000 characters maximum)

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